DERIVED ROW GEOMETRIES

How many different ways are there to derive a tone row? In 12-tone music, a particular ordering of the twelve chromatic pitch classes – the **row** – is used as the basis for a piece. Some composers were interested in ways of generating a 12-tone row from a single **set class**: a collection of unordered pitch-class sets where all sets are the same size and relate to one another by transposition or inversion. In a derived row, a single set class partitions the chromatic scale into discrete, non-overlapping segments. These can be 2-, 3-, 4-, or 6-note segments, depending on the size of the set class. Based on principles of partitions in *modulo* 12 arithmetic, there are finite ways of deriving valid tone rows using certain set classes.

UNDERSTANDING DERIVED ROWS

A chromatic scale consists of 12 pitches that can be represented as a sequence. For example, consider a chromatic scale beginning on C:

C	C♯/D♭	D	D♯/E♭	E	F	F♯/G♭	G	G♯/A♭	A	A♯/B♭	B
0	1	2	3	4	5	6	7	8	9	10 (T)	11 (E)

Pitches considered independently from the octave in which they occur are known as **pitch classes**. Working in *modulo* 12 arithmetic, we can represent each of the 12 pitch classes using a sequence of integers starting with 0.

In 12-tone music, a derived row uses a single set class to partition the tone row into equally-sized chunks. For example, consider the set class **(0, 2)** – the abstract label for the set [0, 2] and all other sets that relate to it by transposition or inversion ([1, 3], [2, 4], and so on). This set class can be used to partition a tone row into 6 chunks of 2 pitch classes. Using the specific set [0, 2] as a starting point, there are two ways of uniquely partitioning the tone row into 6 chunks. We can visualize these as groupings of the twelve pitch classes represented as a clock face:

However, not all partitions are created equal. Some set classes yield only one possible partition of the tone row; others yield several. Furthermore, some set classes are invalid as they cannot partition the tone row perfectly into equally-sized, discrete, non-overlapping segments. Consider the set class **(0, 4)**, which is an invalid partition:

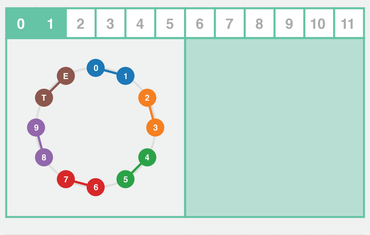
While pitch classes 0 to 7 can partition the tone row using members of this set class, the remaining pitch classes cannot complete a partition without duplicating already-used tones in the tone row. These invalid partitions are a consequence of the cyclical (*modulo* 12) geometry of the tone row, which is clearly represented in the clock face.

This chart shows all possible partition geometries of tone rows for valid set classes, revealing the symmetries and asymmetries of derivation.

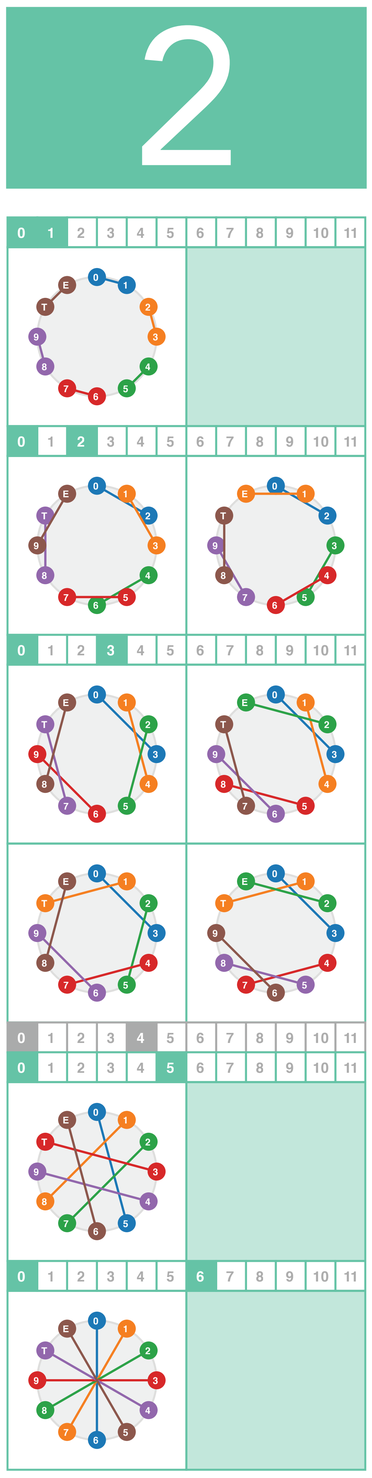
HOW TO READ THIS CHART

Size of set class (number of pitch classes in each set)

Possible set class partition and visualizations of all nonredundant partitions



Invalid set class partition (cannot generate complete derived row)



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This work was created by **Matthew McDonald**, *Associate Professor of Music*, and **Steven Braun**, *Data Analytics and Visualization Specialist*, both at Northeastern University (Boston, MA, USA)